# Material and Energy Balances

## Steady Flow

- fluid continiously passes into and away from system
→ open system

heating power $= \frac{\delta Q}{\delta t}$ mechanical power $= \frac{\delta W}{\delta t}$ mass flow rate $= \frac{\delta m}{\delta t}$

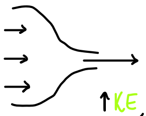
$$\dot{Q} + \dot{W} = \dot{m}\left[\left(h_2 + \tfrac{1}{2}c_2^2 + gz_2\right) - \left(h_1 + \tfrac{1}{2}c_1^2 + gz_1\right)\right]$$

enthalpy K.E P.E

$$\delta Q + \delta W = \delta m\left[\left(h_2 + \tfrac{1}{2}c_2^2 + gz_2\right) - \left(h_1 + \tfrac{1}{2}c_1^2 + gz_1\right)\right]$$

∴ if you have $\Delta h$, KE, PE
→ W + Q done / transfered

### Nozzle
(Accelerates gas)



$\uparrow$KE, $\downarrow \Delta U$ ∴ fluid cools

C = velocity

$\dot{m} = \rho A C$

e.g
3 : 1 diameter
9 : 1 area
1 : 9 velocity
1 : 81 K.E

if relative KE so small for diffuser it can be removed from SFEE

Assume no W or Q transfer

### Diffuser
(decelerates gas)

$$\cancel{\dot{Q}} + \cancel{\dot{W}} = \dot{m}\left[\left(h_2 + \tfrac{1}{2}c_2^2 + \cancel{gz_2}\right) - \left(h_1 + \tfrac{1}{2}c_1^2 + \cancel{gz_1}\right)\right]$$

no change in GPE ≈ (horz.)

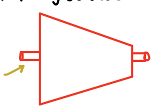
$$\rightarrow h_2 = h_1 + \frac{c_2^2 - c_1^2}{2}$$

$$h = c_p(T - T_0)$$

$$\rightarrow T_2 = T_1 + \frac{(c_2^2 - c_1^2)}{2c_p}$$

### Compressor

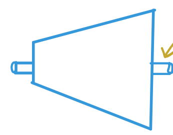
- applies work to raise pressure
→ $\uparrow$T follows




$\boxed{Q = 0}$ ← adiabatic

### Turbine

- Produces useful work from hot, high pressure gas
→ exits cold, low pressure




Shaft Work :
Work done to rotate shaft connected to system : produces torque

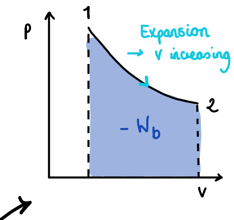
$$\cancel{\dot{Q}} + \dot{W} = \dot{m}\left[\left(h_2 + \tfrac{1}{2}c_2^2 + \cancel{gz_2}\right) - \left(h_1 + \tfrac{1}{2}c_1^2 + \cancel{gz_1}\right)\right]$$

$h = c_p(T - T_0)$

$$\dot{W} = \dot{m}\left[c_p(T_2 - T_1) + \frac{(c_2^2 - c_1^2)}{2}\right]$$

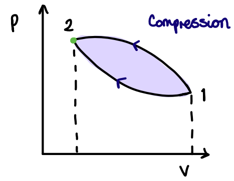
h energy that could be harnessed

K.E losses



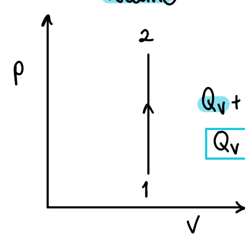

$$W_b = -\int_1^2 p\,dv$$

-ve as it's expansion
→ W done on surroundings




Difference in work
Same final state, different work
independent

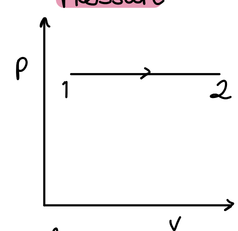
### Constant Volume (Isochoric)




$$Q_v + \cancel{W} = \Delta U$$

$$\boxed{Q_v = \Delta U}$$

### Constant Pressure (Isobaric)




$$-p\int_1^2 dv = \boxed{-p(V_2 - V_1)}$$

constant

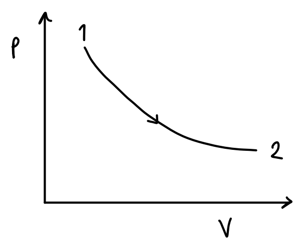
$Q_p = \Delta U - W$

$Q_p = \Delta U + p\Delta V$

$Q_p = \Delta(U + pV)$

$\boxed{Q_p = \Delta H}$

### Constant Temperature (Isothermal)




$\Delta U = 0$

$0 = Q - W$

$Q = W$

$$W = -\int_1^2 p\,dv = -mRT\frac{dv}{V}$$

constants

$$= -mRT\int_1^2 \frac{dv}{V}$$

$$= \boxed{-mRT\ln\left(\frac{V_2}{V_1}\right)}$$

$$= -p_1V_1\ln\left(\frac{V_2}{V_1}\right)$$

$$= -p_2V_2\ln\left(\frac{V_2}{V_1}\right)$$

obtained as $\int_{V_1}^{V_2}\frac{dv}{V} = \int_{V_1}^{V_2}\frac{1}{V}\,dv = [\ln V]_{V_1}^{V_2}$

$= \ln V_2 - \ln V_1$

$= \ln\left(\frac{V_2}{V_1}\right)$

### Polytropic Process $(pV^n)$

(interchange of both work and heat between system and surroundings)

$$\boxed{pV^n = \text{constant}}$$

$$p_1V_1^n = p_2V_2^n$$

$$W_b = \frac{p_2V_2 - p_1V_1}{n - 1}$$

and

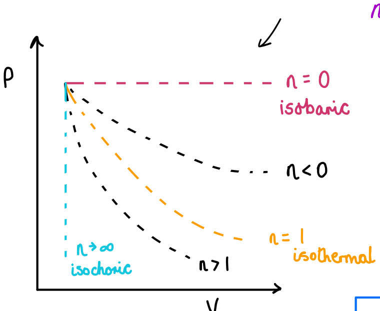
$$W_b = \frac{mR(T_2 - T_1)}{n - 1}$$

Using ideal gas law equations and $pV^n$ = constant

$\left(\frac{mRT}{V}\right)V^n$ = constant

$TV^{n-1}$ = constant

→ T = constant $p^{\left(\frac{n-1}{n}\right)}$




$n = \gamma$
isentropic

any other n not isothermal, $T_2 \neq T_1$
$Q \neq 0$, $W \neq 0$

### Isentropic Process $(pV^\gamma)$

→ No friction
→ No heat transfer (adiabatic)

$$W = U_2 - U_1 - \cancel{Q}$$

$$\boxed{W = mc_v(T_2 - T_1)}$$

Replacing T with p and V from $T = \frac{pV}{mR}$

$$W = \frac{m}{m}\frac{c_v}{R}(p_2V_2 - p_1V_1)$$

$$\frac{c_v}{R} = \frac{c_v}{c_p - c_v} = \frac{1}{\frac{c_p}{c_v} - 1}$$

$$\boxed{\frac{c_p}{c_v} = \gamma} \rightarrow \frac{c_v}{R} = \frac{1}{\gamma - 1}$$

$$\boxed{W = \frac{p_2V_2 - p_1V_1}{\gamma - 1}}$$

$\gamma$ = specific capacity ratio

$$\boxed{pV^\gamma = \text{constant}}$$

$$p = CV^{-\gamma}$$

$$W_b = -\int_1^2 CV^{-\gamma}\,dV$$

$$W_b = -\frac{\left[CV^{-\gamma+1}\right]_1^2}{1-\gamma}$$

$$W_b = \frac{\left[(CV^{-\gamma})V\right]_1^2}{\gamma - 1} = \frac{[pV]_1^2}{\gamma - 1} = \frac{p_2V_2 - p_1V_1}{\gamma - 1}$$